5-5 Videos Guide

5-5a

- The del operator
    - $\nabla = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$
- The curl of a vector field $\mathbf{F}(x, y, z) = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$
    - $\text{curl } \mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$

Theorem (statement and proof):

- If $\mathbf{F}$ is conservative, then $\text{curl } \mathbf{F} = \mathbf{0}$ (so if $\text{curl } \mathbf{F} \neq \mathbf{0}$, then $\mathbf{F}$ is not conservative)

Theorem (statement):

- The converse of the above theorem is true if $\mathbf{F}$ is defined on all of $\mathbb{R}^3$

5-5b

- The divergence of a vector field $\mathbf{F}(x, y, z) = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$
    - $\text{div } \mathbf{F} = \nabla \cdot \mathbf{F} = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{d}{dz} \rangle \cdot \langle P, Q, R \rangle = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$

Exercise:

- Find (a) the curl and (b) the divergence of the vector field.
  $\mathbf{F}(x, y, z) = x^3yz^2 \, \mathbf{j} + y^4z^3 \, \mathbf{k}$

- Zero results
    - If $\text{curl } \mathbf{F} = \mathbf{0}$ at P, then $\mathbf{F}$ is said to be *irrotational* at P.
    - If $\text{div } \mathbf{F} = 0$, then $\mathbf{F}$ is said to be *incompressible*.

Theorem (statement and proof):

- If $\mathbf{F} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$ and P, Q, and R have continuous second-order partial derivatives, then $\text{div curl } \mathbf{F} = \mathbf{0}$.

5-5c

- Vector forms of Green's Theorem
    - $\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_D (\text{curl } \mathbf{F}) \cdot \mathbf{k} \, dA$
    - $\oint_C \mathbf{F} \cdot \mathbf{n} \, ds = \iint_D \text{div } \mathbf{F}(x, y) \, dA$